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A FUZZY EVALUATION MODEL OF MANUFACTURING MACHINERY IN TERMS OF SUSTAINABLE BUSINESS*

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Abstract. The inclusion of sustainability criteria in the investment decision processes requires application of tools that consider the vague nature of sustainable characteristics. Fuzzy approach can process the vagueness of sustainability measures. The aim of the paper is to develop a suitable model of the fuzzy assessment of investment alternatives measured by Sustainability Utility Index. Its scheme is described methodologically as a three-stage system of multi-criteria ex-ante evaluation. An algorithmic conception of the system is performed at its third stage by a "fuzzy processor". The fuzzy model is applied to the task of the optimal type of building machinery selection in view of the cost and sustainable criteria. The results enable the manager to optimize his/her decision, who aims to minimize the investment and operation costs of the building machinery. Compared to the fuzzy approach based on Mandani´s system, our model avoids several complications associated with the semantics of fuzzy implications in various fuzzy logics, it is more user friendly and easily algorithmized. This model is thus suitable to apply not only in corporate sphere but also in areas of ecology, sociology, economics. The limits of the fuzzy system are twofold - on the general level the fuzzy systems lack the ability to learn and memorize; determining or tuning a suitable membership function and fuzzy rules is not always clear. The challenge is first to test the model in the corporate sphere and second to acquaint potential users with this computationally undemanding fuzzy procedure.

Keywords: multi-criteria decision-making; corporate production; ex-ante evaluation; fuzzy processor; fuzzification table; sustainability utility index

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JEL Classifications: D25, E22, G11, G31

1. Introduction to the issue of sustainability in corporate decision-making

A socially responsible company assumes responsibility for the impacts that result from its activities. It integrates environmental, social and ethical views, consumer interests and human rights into its strategy and business activities (Clarke & Boersma, 2017; Pan et al., 2022; Skare & Porada-Rochon, 2022). Corporate social

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responsibility is based on the same pillars as sustainable development - both concepts are based on the same basic principle of the Triple Bottom Line (Hammer & Pivo, 2017). Business practice shows that most companies strive to promote the principles of social responsibility and sustainability in order to improve the image, reputation and brand. Good reputation increases stakeholder confidence in society (Kim, 2019; Gavurova et al., 2022; Belas et al., 2022). This goes hand in hand with growing customer satisfaction and loyalty, which can have a positive impact on economic results (Eklof et al., 2020). Businesses are responding to the shift in thinking of modern customers towards the principles of sustainable development (e.g., the number of consumers who take into account whether the product is eco-labelled, etc.) and trying to meet their expectations (Gold & Schleper, 2017). According to Nadanyiova & Das (2020), companies promoting corporate social responsibility are driven mainly by millennials, who prefer brands that use this approach as a means of communication with this group of customers.

Sustainability of production has become a global problem, especially in developed and rapidly developing countries (Yu et al., 2022; Xu et al., 2021; Qin et al., 2021; Susanto et al., 2017). An improved corporate sustainability performance does not automatically lead to an improved sustainability in the systems in which the company operates (Wyborn et al., 2019). Thus, it is necessary to identify a comprehensive evaluation framework in order to analyse whether an improving corporate sustainability can make a positive contribution to sustainable development in a broader sense. This includes first and second level performance of assessments (Awasthi et al., 2018). At the first level, direct impacts on sustainability are assessed, focusing on efficiency issues, while sustainability performance in a broader sense focuses on systemic efficiency and covers the impacts of sustainability on society and nature as a whole. This problem is often solved as a multicriteria optimization problem (Franciosi et al., 2018).

Incorporating principles of sustainability in the business decision-making processes requires the application of tools that can cover the vague concept of sustainable measures (e.g., quality of the relationship with the surrounding community, various forms of social and moral obligations, etc. – see Rostamnezhad et al., 2020). Karkalíková & Strhan (2018) analyzed the factors that affect the company's overall performance and also determined what factors can cause the company certain problems. This article focuses on a procedure by means of which a decision-maker, which takes into account vaguely defined parameters of sustainable development relevant to the company activities, reaches a rational solution. This solution should be an acceptable compromise of the requirements of all parties involved, i.e., the requirement to ensure the increase of the value of the company and to reflect the responsibility for the consequences of the company's activities.

The central aim of the paper is to develop a three-stage model of the fuzzy assessment of a sustainable production in terms of its cost indicators and vague sustainable characteristics. Within it we ask the question: RQ1 “Does the fuzzy model of multi-criteria evaluation have the property to state a rational decision from the point of view of stakeholders and principals of sustainability?”. Furthermore, we ask: RQ2 “Is there an intersection of agreement as well as differences in demands between the here presented fuzzy procedure and other model/models based on fuzzy logic?”.

The paper is structured to the review, methodological and practical part. The review part captures a selected overview of techniques for decision-making utilized in the case of vague and uncertain entry data. Methodology provides the insight to the basic principles of the fuzzy approach. This technique is applied to the solution of a machinery selection based on the cost and vague input parameters. Conclusion part summarises the most important points and discusses results and findings.

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2. Review part

Problems with understanding sustainability, and its modelling, evaluating and managing, falls under the so-called "wicked problems" (Lönngren & Van Poeck, 2021). One of the main challenges in the sustainability issues, which addresses wicked problems, is the existence of uncertainty. To understand the tools that can assist the decision-makers under uncertainty is thus necessary (Bogachov et al., 2020). Starting from heuristics to the modern computational tools of dealing with uncertainty is a vast area of research. Let us mention e.g., Ahi et al. (2018) who proposed an original probabilistic weighting model for systematically allocating priorities in sustainability measurement; Mishra et al. (2020) who applied sensitivity analysis in order to get optimal feasible solution that validate the sustainable economic production; Ladu & Morone (2021) who introduced the scoring tool consisting of 48 sustainability indicators with proposed metrics based on existing standards, methodologies and best practices in sustainability assessment. The artificial neural networks were used by Vochozka et al. (2019) to analyse the sustainable investments in the capital market. The data envelopment analysis was proved to be an appropriate approach to select efficient sustainable projects (Fiala, 2018). The sustainability of a company's performance is largely dependent on the company's employees. Kovářiková et al. (2021) considered setting up an incentive system for the company's employees, which leads to maintaining or even increasing their performance as well as the performance of the whole company. Lavičková et al. (2021) identified the possibility of using subsidized language courses by the employer as the main motivational benefit for maintaining / increasing employee performance.

Fuzzy logic is an approach that has been effectively used to decide within sustainability assessments made under uncertainty (Ziyadin et al., 2019; Kelemen et al., 2022; Gavurova et al., 2022). It has been proved that it closely approximates human decision-making and perception processes. Fuzzy logic enables the researches to normalize quantitative and qualitative sustainability indicators and evaluate indicators with a vague definition (Zarte et al., 2018; Skare et al., 2023 a,b). In this fuzzy spirit we briefly review some of the fuzzy techniques utilized in managerial problems regarding sustainable investment decisions.

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) developed by Hwang & Yoon (1981) and Intuitionistic Fuzzy Sets mathematically defined by Atanassov (1994) are applied in Onat et al. (2016) for a hybrid life cycle sustainability assessment for different conventional and alternative vehicles technologies. Expert judgement is projected to weight setting and applied to each dimension of different alternatives.

Zarte et al. (2018) present a concept for a fuzzy inference model to evaluate short- and mid-term production planning programs taking into account sustainable indicators. The formulation of the model lies upon common methods and fuzzy operators from the fuzzy set theory. As the authors state, the research can specifically support decision-making in e.g., decreasing production costs, saving resources, and improving employees' wellbeing. The drawback can be seen in not testing the functionality of the designed concept in practical use.

Wang (2019) used the fuzzy c-means model to predict the financial distress of companies listed on the Shenzhen and Shanghai Stock Exchange. Using the model used, he identified a total of 11 financial indicators, which can be used to predict the risk of financial distress with very different prediction results.

Hašková (2016) applied fuzzy logic to the evaluation of partial components of the business environment in selected European and Asian countries. These were levels of corruption, economic and political stability.

Boloș et al. (2019) propose a fuzzy logic tool for the assets acquisition. The algorithm lies on three main components - the matrix of the membership degree as fuzzy triangular numbers, the vector of the global membership degree, the maximum of the global membership degree. The result of two scenarios test of asset acquisition states that the acquisition cost of the assets can be well combined with their economic performance.

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Amiri (2010) proposes a new methodology to provide an approach for assessing alternative projects to assist a decision-maker to select the best candidate. Within the AHP and fuzzy TOPSIS techniques author utilizes linguistic criterial preferences. The AHP (whose algorithm was developed by Ateş et al., 2006, who justified its application in the performance evaluation of a faculty in any department of any university) is used for analysing the structure of the problem and determining weights of the criteria. The fuzzy TOPSIS method (a general view of which development is precisely given by Nădăban et al., 2016) is used to obtain final ranking. The analysis confirms the importance of correctly set criteria weights in fuzzy TOPSIS method (also dealt with e.g. in Memari et al., 2019).

Rajesh (2019) proposes the Resilient Fuzzy Index for measuring the level of resilience of firms and the Performance Fuzzy Index that aids in identifying critical attributes affecting resilience in supply chains. The indexes could help the management in evaluating resilience capabilities of the supply chain and simultaneously include sustainability requirements within the decision making (Horák et al., 2020).

Jankova et al. (2021) performs an easy-to-use model containing input variables that fundamentally influence the investment decision in the US stock market. The study uses fuzzy Sugeno inference system T2FLS type. The T2FLS is then compared, based on statistical measures RMSE, R2, MAE, MAPE, with the T1FLS type of both Sugeno and Mamdani. The authors prove that compared to type-1 fuzzy logic using type-2 fuzzy logic leads to more realistic and accurate results. Authors justify the higher performance of type-2 fuzzy logic system mainly due to the three-dimensional membership function of the general type-2 fuzzy set (also confirmed in Muljono et al., 2018).

Sugeno fuzzy inference system is regarded as an alternative to Mamdani's system - both are often subjected to comparison (e.g., Saleh et al., 2017; Chaudhary, 2019). Comparing Sugeno´s and Mamdani´s defuzzification process, the Mamdani´s is more computationally efficient, as it uses a weighted average or weighted sum of a few data points rather than compute a centroid of a two-dimensional area (Chaudhari, 2014). Janková & Dostál (2021) justifies the usage of the fuzzy inference system of the Mamdani type for the model construction to support investment decisions in Exchange-Traded Funds as it operates better with unstructured or poorly structured input data than the Sugeno type.

Amaral et al. (2019) propose the use of Computational Intelligence algorithms to predict cryptocurrencies values based on historical values. The Mamdani system is applied to suggest what to do with the invested value. Experiments show that the proposed approach is promising and competitive with alternatives reported in the literature.

When dealing with sustainable "fuzzy" issues many articles prefer the Mamdani ´s approach. As an example, let us mention Hendiani & Bagherpour (2019) who evaluated a sustainable construction, which ought to enhance the quality of social, environmental, and economic practices by stating the sustainability level and identifying the weaknesses and improving them. The reasons lay in the benefits that Mamdani´s method offer: intuitiveness, suitability to human inputs, interpretability of the rule base, etc.

3. Methodology: the fuzzy approach

Let the set U be a field of consideration or discussion (universe). Let $\mu_{\underline{A}}: U \rightarrow \langle 0,1 \rangle$ be a membership function and let $\underline{A} = \{(y, \mu_{\underline{A}}(y)): y \in U\}$ be the set of all pairs $(y, \mu_{\underline{A}}(y))$, in which the number $0 \leq \mu_{\underline{A}}(y) \leq 1$ states the degree of membership of the pair $(y, \mu_{\underline{A}}(y))$ to the set $\underline{A}$ on the given $y \in U$. Then $\underline{A}$ is a *fuzzy subset* on the universe U. The significant characteristic of the fuzzy subset $\underline{A}$ is its support $U_{\underline{A}} = \{y: 0 < \mu_{\underline{A}}(y) \leq 1, y \in U\} \subset U$. In terms of fuzzy logic $\mu_{\underline{A}}(y) = \lfloor y \in U_{\underline{A}} \rfloor$, where $\lfloor y \in U_{\underline{A}} \rfloor$, denotes the degree of veracity of the proposition that y is the element of the

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support of the fuzzy set $\underline{A}$. The element $y \in U$ with $\mu_{\underline{A}}(y) = 0.5$ is called the *crossover point* in $\underline{A}$. Values that are greater than 0.5 signal that the element y rather belongs to $U_{\underline{A}}$, the values smaller suggest it rather does not belong to it (the details in e. g., Běhounek & Daňková, 2016; Padilla-Rivera et al., 2021).

The fuzzy subset $\underline{A}$ (whose support $U_{\underline{A}} \subset U \subset R$, where R is the set of real numbers, and its function $\mu_{\underline{A}}$ is gifted by the property of normality and convexity (i.e., at least in the case of one element $x \in U_{\underline{A}}$ it applies $\mu_{\underline{A}}(x) = 1$, and $\mu_{\underline{A}}(x') \geq \min\{\mu_{\underline{A}}(x_1), \mu_{\underline{A}}(x_2)\}$ for all $x' \in \langle x_1, x_2\rangle \subset U_{\underline{A}}$) is called the *fuzzy number* (Torkabadi et al., 2018). The fuzzy numbers are also formal models of linguistic terms (i.e., expressed in natural language) of variables occurring in managerial decision-making tasks (see e.g., Holčapek et al., 2021).

Linguistic variables thus acquire their values at two levels: at the linguistic level and the numerical level. At the linguistic level they are usually performed by terms (fuzzy numbers) of the type, e.g., the low value ($\underline{L}$), common value ($\underline{M}$) and high value ($\underline{H}$); at the numerical level they are represented by the real numbers from the interval $U = \langle 0, 100\rangle$ (in detail, e.g., Hašková & Fiala, 2019).

The relationship between the two mentioned levels of values of the linguistic variable is evident from the fuzzification Table 1. It defines the projection of μ: $\{\underline{L}, \underline{M}, \underline{H}\} \times U \rightarrow \langle 0,1\rangle$ in the form $\mu(\underline{T}, u) = \mu_{\underline{T}}(u)$, which is based on the model with one internal and two border fuzzy sets for the terms *low* ($\underline{L}$), *common* ($\underline{M}$), and *high* ($\underline{H}$). Interval U is thus divided with the constants a, b, c, d into five sections and described by the membership functions (1), in which a, b, c, d $\in \langle 0, 100\rangle$, $0 \leq a \leq b \leq c \leq d \leq 100$ are given by an expert:

$$
\begin{array}{ll}
(\underline{L}) & \begin{array}{l} \mu_{\underline{L}}(y) = 1 \text{ for } y < \mathrm{a}, \\ \mu_{\underline{L}}(y) = (\mathrm{b} - y) \,/\, (\mathrm{b} - \mathrm{a}) \text{ for } \mathrm{a} \leq y < \mathrm{b}, \\ \mu_{\underline{L}}(y) = 0 \text{ otherwise.} \end{array} \\
(\underline{M}) & \begin{array}{l} \mu_{\underline{M}}(y) = (y - \mathrm{a})/(\mathrm{b} - \mathrm{a}) \text{ for } \mathrm{a} \leq y < \mathrm{b}, \\ \mu_{\underline{M}}(y) = 1 \text{ for } \mathrm{b} \leq y < \mathrm{c}, \\ \mu_{\underline{M}}(y) = (\mathrm{d} - y)/(\mathrm{d} - \mathrm{c}) \text{ for } \mathrm{c} \leq y < \mathrm{d}, \\ \mu_{\underline{M}}(y) = 0 \text{ otherwise.} \end{array} \\
(\underline{H}) & \begin{array}{l} \mu_{\underline{H}}(y) = 0 \text{ for } y < \mathrm{c}, \\ \mu_{\underline{H}}(y) = (y - \mathrm{c}) \,/\, (\mathrm{d} - \mathrm{c}) \text{ for } \mathrm{c} \leq y < \mathrm{d}, \\ \mu_{\underline{H}}(y) = 1 \text{ otherwise.} \end{array}
\end{array} \tag{1}
$$

Table 1. Fuzzification table

Interval	***u*** **< a**	**a ≤ *u* < b**	**b ≤ *u* < c**	**c ≤ *u* < d**	***u*** **≥ d**
L	1	(b – u) / (b – a)	0	0	0
M	0	(u – a) / (b – a)	1	(d – u) / (d – c)	0
H	0	0	0	(u – c) / (d – c)	1

Source: own processing

The inner nonzero fields of the fuzzification table define one-element or two-element subset $h(u) = \{\underline{T}: \underline{T} \in \{\underline{L}, \underline{M}, \underline{H}\}, \mu(\underline{T}, u) > 0\}$ of the set $\{\underline{L}, \underline{M}, \underline{H}\}$for each $u \in U$.

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4. The formulation of the three-stage fuzzy system of multicriteria alternative evaluation

The scheme of the three-stage fuzzy system of multicriteria ex-ante evaluation is introduced in Fig. 1. The basic functions of all elements of the fuzzy system structure (the right scheme in Fig. 1) corresponds with the case of ex-post evaluation (certainty on the inputs side, where only specific point values enter the system at the first stage). The point values are the outputs of the criterion functions at the second stage entering the third stage of the system (block K in Fig. 1). The number $v \in \langle 0, 100\rangle$ on output side of the block K is the numerical value of the output linguistic variable of the ex-post evaluated alternative.

4.1 The three-stage fuzzy system of multicriteria alternative for ex-ante evaluation

In the case of ex-ante evaluation, the fuzzy system appears in the situation of uncertainty on the input data side. At its first stage, only the uncertain data characterized by intervals of values of their possible occurrences or mixes of uncertain and numerical data enter the solving process (in Fig. 1 the intervals $\langle a_{1\min}, a_{1\max}\rangle \ldots \langle a_{C\min}, a_{C\max}\rangle$ to $\langle ep_{\min}, ep_{\max}\rangle$). This external uncertainty is then transferred by the criterion functions to the internal uncertainty of the inputs to block K (see intervals $\langle x_{1\min}, x_{1\max}\rangle \ldots \langle x_{N\min}, x_{N\max}\rangle$ to $\langle ae_{\min}, ae_{\max}\rangle$ at the second stage of the model), as well as to its output (see interval $\langle v_{\min}, v_{\max}\rangle$). The right picture of Fig. 1 shows a case where some external uncertain data enter block K directly. They usually represent subjective quantification of qualitative data expressed by intervals at the value scale from 0 to 10 or from 0 to 100.

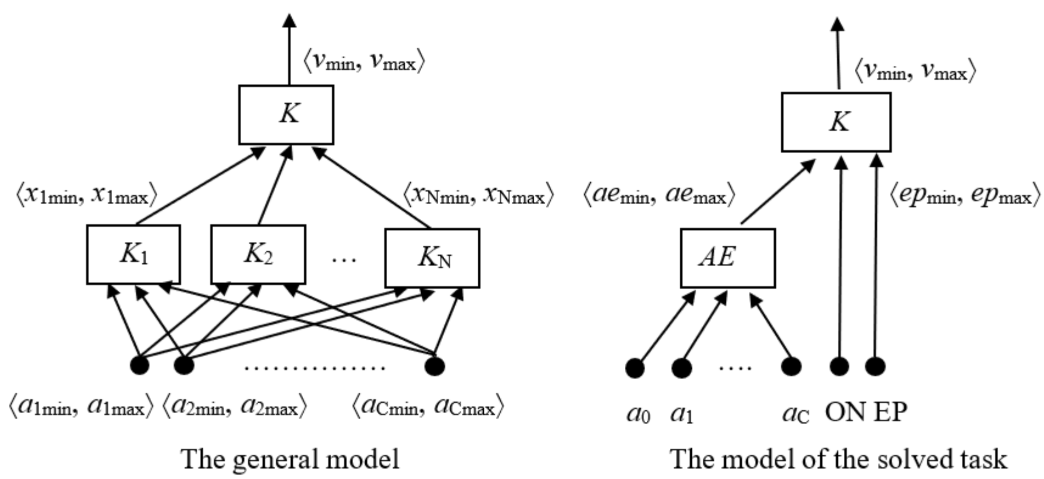

Figure 1. Structure of the three-stage fuzzy system of multicriteria ex-ante evaluation

Source: own processing

In the first step the conversion of external uncertainty to internal uncertainty is carried out. For this purpose, the relations defining the individual criterion functions are translated into the language of interval algebra, defined and discussed by (Yuan et al., 2009). The input intervals are converted to output intervals by applying interval algebra operations. At the second stage of the model, N intervals are created that are presented by at most two-element sets of their extreme values (in the case of a mix of uncertain and numerical data, the numerical value is considered as an interval with two identical extreme values). The Cartesian product can result in the creation of up to 2^N N-dimensional numerical vectors entering the block K. By the gradual processing of each of them by the computational fuzzy algorithm, whose block scheme is in Fig. 2, a set of partial calculation results (numbers v) is calculated. The arithmetic mean $y = (v_{\min} + v_{\max}) / 2$ of the minimum and maximum of this set is then the

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subjectively expected numerical value of the output linguistic variable *SUI* (Sustainability Utility Index) of the evaluated alternative. It performs the sustainable social costs.

4.2 Computational algorithm of the fuzzy process

The procedures by which the fuzzy system (block *K*) processes its numerical inputs are analogous to the procedures by which the human mind processes visual, auditory, tactile and other stimuli and generates corresponding responses.

The N-dimensional vector $(x_1,\ldots,x_N)$ entering the block *K* is transformed into a vector $(u_1,\ldots,u_N)$ by converting the coordinates x_i into a scale in the range 0 to 100. Its coordinates u_i, i = 1,…, N, are included in the adequate input fuzzy sets; together they select suitable inference rules for their manipulation, define their "strength" and generate the membership function μ_{agg} on the field $V = \langle 0, 100\rangle$ of the output linguistic variable *SUI*. The horizontal coordinate of the centre of gravity below its course $\mu_{agg}(v)$ is the result of the calculation.

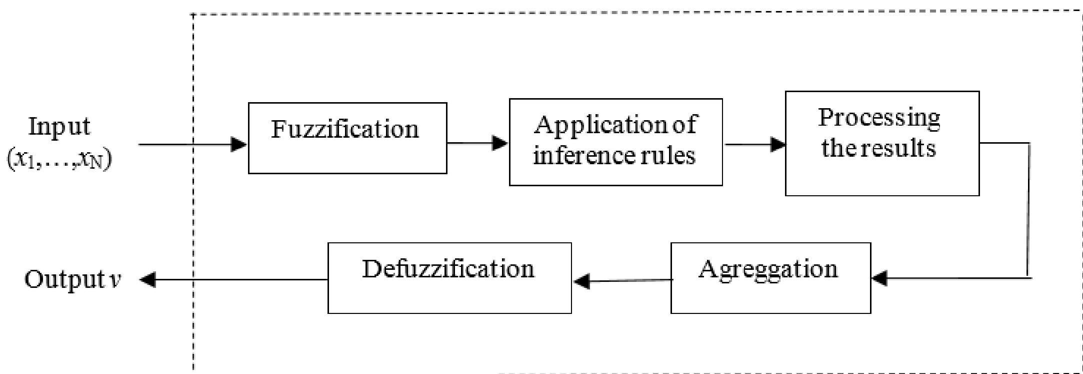

Figure 2. The continuity diagram of individual phases of the fuzzy approach of the computational algorithm

Source: own processing

In the fuzzification phase, the expected influence of the x_i coordinate of the vector $(x_1,\ldots,x_N)$ on the output v is first taken into account. If positive, x_i is converted to the *internal* value u_i according to the formula $u_i = 100 \cdot (x_i - Xi_{min}) / x_{iref}$, $x_{iref} = Xi_{max} - Xi_{min}$, where $\langle Xi_{max}, Xi_{min}\rangle$ is a domain of numerical values of a linguistic variable on the i-*th* input. If the effect is negative, this formula $u_i = 100 - 100 \cdot (x_i - Xi_{min}) / x_{iref}$ for the conversion is used. If the positive effects prevail, the resulting *SUI* has the character of a contribution, if the negative effects prevail, it has the character of a harm (cost).

Each i-*th* input, i = 1 to N, has its own fuzzification table, which generates its own set $h(u_i) = \{\underline{T}_i: \underline{T}_i \in \{\underline{L}_i, \underline{M}_i, \underline{H}_i\}, \mu(\underline{T}_i, u_i) > 0$. The Cartesian product $H = h(u_1)\times\ldots\times h(u_N) = \{(\underline{T}_1,\ldots,\underline{T}_N): \underline{T}_1 \in h(u_1),\ldots,\underline{T}_N \in h(u_N)\}$ with 2^α elements is formed from them, where α, $0 \le \alpha \le N$ is the number of two-element sets $h(u_i)$ in the Cartesian product *H*. The interference rule is the element of the projection p: $\{\underline{L}_1, \underline{M}_1, \underline{H}_1\}\times\ldots\times\{\underline{L}_N, \underline{M}_N, \underline{H}_N\} \longrightarrow \{\underline{L}, \underline{M}, \underline{H}\}$, where $\underline{L}$, $\underline{M}$ and $\underline{H}$ are the terms of the output linguistic variable *SUI* with a domain of numerical values $V = \langle 0, 100\rangle$. The set of inference rules thus consists of a total 3^N pairs of the type $((\underline{T}_1,\ldots,\underline{T}_N), T)$, which are formulated by a knowledgeable expert. In the phase of inference rules application, three classes are created $H(\underline{T}) = \{(\underline{T}_1,\ldots,\underline{T}_N): (\underline{T}_1,\ldots,\underline{T}_N) \in H \cap p^{-1}(\underline{T})\}$, $\underline{T} = \underline{L}, \underline{M}, \underline{H}$ of decomposition of the set *H* according to the terms of the output linguistic variable *SUI*, where the relation p^{-1} is the inversion of the projection *p*. To each in such a way created class $H(\underline{T})$ its characteristic number $M_{\underline{T}} \in \langle 0, 1\rangle$ is then assigned:

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- if the class is empty ($H(\underline{T}) = \emptyset$), then $M_{\underline{T}} = 0$;
- if $H(\underline{T}) \neq \emptyset$, then $M_{\underline{T}} = \max\{\{\min\{\mu(\underline{T}_1, u_1),\ldots, \mu(\underline{T}_N, u_N)\}: (\underline{T}_1,\ldots,\underline{T}_N) \in H(\underline{T})\}\}$.

In the processing phase of results, the fuzzy algorithm by means of the numbers $M_{\underline{L}}$, $M_{\underline{M}}$ and $M_{\underline{H}}$ restricts (cuts off) the courses of the functions $\mu_{\underline{L}}(v)$, $\mu_{\underline{M}}(v)$ and $\mu_{\underline{H}}(v)$ of the terms $\underline{L}$, $\underline{M}$ and $\underline{H}$ of the output linguistic variable *SUI* on the domain of its numerical values $V = \langle 0, 100\rangle$ from the above - see Fig. 3 on the left. Then, in the aggregation phase, the fuzzy algorithm fuzzy logically adds their torso, by means of which it aggregates them into the resulting function $\mu_{agg}(v)$, which creates their "wrapping" - see Fig. 3 on the right.

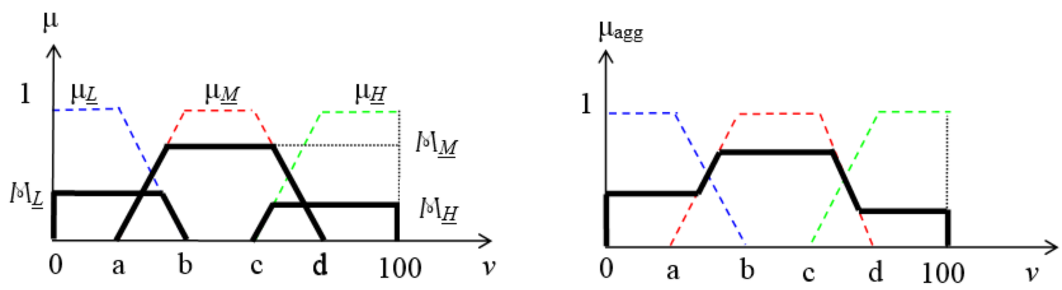

Figure 3. Example of cutting off the functions $\mu_{\underline{L}}(v)$, $\mu_{\underline{M}}(v)$, $\mu_{\underline{H}}(v)$ and their aggregation into the resulting $\mu_{agg}(v)$

Source: own processing

In such a way formed $\mu_{agg}(v)$ can be included in one of the seven classes based on mutual relationships between the characteristic numbers $M_{\underline{L}}$, $M_{\underline{M}}$ and $M_{\underline{H}}$, as depicted in the final component of the flowchart of the algorithm of fuzzy process (see Fig. 4) by means of the blocks $\mu = \mu_{I}$ to $\mu = \mu_{VII}$. In the case of the example shown in Fig. 3 it follows that the path takes us through its "maze" to the class $\mu = \mu_{VI}$. For each of these classes, a general analytical description of the function $\mu_{agg}(v)$ is given, including formulas for calculating the values of its certain integrals $\Delta = \int v\cdot\mu_{agg}(v)\cdot dv$ and $\Gamma = \int\mu_{agg}(v)\cdot dv$, which are part of the defuzzification phase and whose division is the result of the calculation.

Specifically, for the class $\mu = \mu_{VI}$, to which the example in Fig. 3 belongs, the following applies:

$\mu_{agg}(v) = M_{\underline{L}}$ for $v < a + (b - a)\cdot M_{\underline{L}}$,
$= (v - a) / (b - a)$ for $a + (b - a)\cdot M_{\underline{L}} \leq v < a + (b - a)\cdot M_{\underline{M}}$,
$= M_{\underline{M}}$ for $a + (b - a)\cdot M_{\underline{M}} \leq v < c + (d - c)\cdot(1 - M_{\underline{M}})$,
$= (d - v) / (d - c)$ for $c + (d - c)\cdot(1 - M_{\underline{M}}) \leq v < c + (d - c)\cdot(1 - M_{\underline{H}})$,
$= M_{\underline{H}}$ for $c + (d - c)\cdot(1 - M_{\underline{H}}) \leq v$.

$\int\mu_{agg}(v)\cdot dv = (a + (b - a)\cdot M_{\underline{L}})\cdot M_{\underline{L}} +$
$(b - a)\cdot(M_{\underline{M}}^2 - M_{\underline{L}}^2) / 2 +$
$M_{\underline{M}}\cdot((c - a) + (d - c)\cdot(1 - M_{\underline{M}}) - (b - a)\cdot M_{\underline{M}}) +$
$(d - c)\cdot(M_{\underline{M}}^2 - M_{\underline{H}}^2) / 2 +$
$M_{\underline{H}}\cdot(100 - c - (d - c)\cdot(1 - M_{\underline{H}}))$.

$\int v\cdot\mu_{agg}(v)\cdot dv = (a + (b - a)\cdot M_{\underline{L}})^2\cdot M_{\underline{L}} / 2 +$
$(b - a)\cdot(a\cdot(M_{\underline{M}}^2 - M_{\underline{L}}^2) / 2 + (b - a)\cdot(M_{\underline{M}}^3 - M_{\underline{L}}^3) / 3) +$
$M_{\underline{M}}\cdot((c + (d - c)\cdot(1 - M_{\underline{M}}))^2 - (a + (b - a)\cdot M_{\underline{M}})^2) / 2 +$
$(d - c)\cdot(c\cdot(M_{\underline{M}} - M_{\underline{H}}) + (d/2 - c)\cdot((1 - M_{\underline{H}})^2 - (1 - M_{\underline{M}})^2) - (d - c)\cdot((1 - M_{\underline{H}})^3 - (1 - M_{\underline{M}})^3) / 3) +$
$M_{\underline{H}}\cdot(5\,000 - (c + (d - c)\cdot(1 - M_{\underline{H}}))^2 / 2)$.

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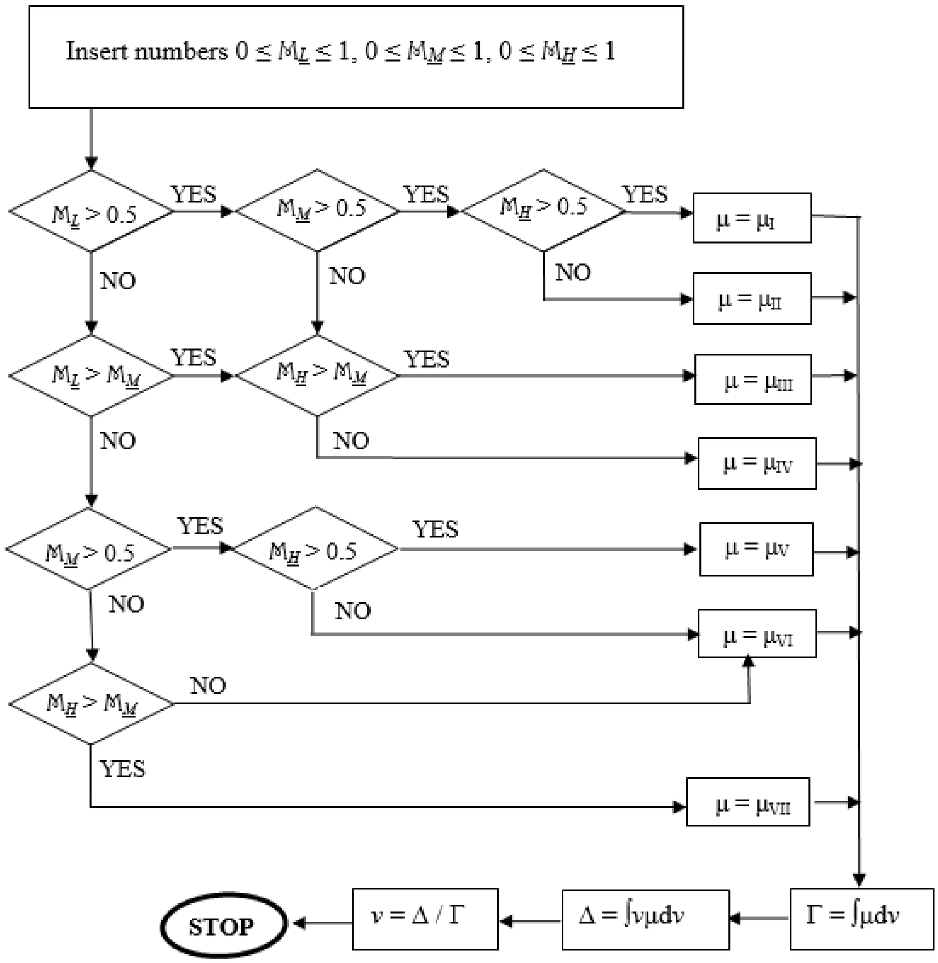

Figure 4. The flowchart of the fuzzy process algorithm

Source: own processing

5. Application

The fuzzy technique is applied to the optimal solution of a building machinery selection based on the cost and vague input parameters. Several ways exist to calculate the expected optimal life of a machinery. These methods incorporate the purchase price, annual operating costs and annual operating profits, and final value. It is assumed for the machinery to gradually lose productivity and/or the gradual increase of operating costs. The building company evaluative framework works both at the production system grade and the environmental and society grade. The task is to assess a purchase of a building machinery.

5.1 Database

The source estimates are based on the data research from the Albertina database (2020) and data research and estimates in the Czech machinery market aimed for building construction that was conducted by the authors with cooperation of experts of the building companies. The research resulted in four alternative variants that poses the technological requirements; in the next we denote them machinery A, B, C, D – Table 2.

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The revenues from the expected annual sale of services generated by the machinery employment do not depend on the contemplated variant. The optimality of the choice is thus assessed according to the economic, environmental, and social criteria: annual expenses (A), machine operation noise (ON) and environmental pollution (EP). These criteria perform economic, environmental and social costs for which the lowest value is desirable.

The entry data are presented by Table 2. The purchase price of the machineries and the economic lifetime are certain data. Other input data are uncertain, for which only estimates of intervals are available, i.e., A, ON and EP.

Table 2. The purchase price and intervals of possible values of annual expenses (A) in thousands EUR; intervals of values of machine operation noise (ON) and environmental pollution (EP)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**ON**	**EP**
Machinery A	31,0	⟨7, 9⟩	⟨11, 14⟩	⟨16, 20⟩	⟨24, 30⟩			⟨20, 40⟩	⟨50, 70⟩
Machinery B	42,5	⟨5, 6⟩	⟨8, 10⟩	⟨12, 15⟩	⟨16, 20⟩	⟨20, 25⟩	⟨24, 30⟩	⟨60, 70⟩	⟨60, 80⟩
Machinery C	57,6	⟨4, 5⟩	⟨7, 9⟩	⟨10, 12⟩	⟨14, 18⟩	⟨19, 22⟩		⟨30, 50⟩	⟨40, 60⟩
Machinery D	62,3	⟨3, 4⟩	⟨6, 8⟩	⟨9, 11⟩	⟨12, 14⟩	⟨16, 20⟩	⟨23, 31⟩	⟨60, 70⟩	⟨50, 70⟩

Source: own processing

The column 0 states the machinery purchase price. The machineries differ with their economic operation life that ranges between 1 – 6 years and intervals of possible values of annual operation expenses (columns 1 – 6). In Fig. 1 on the right, the operation expenses are given as the symbols a_0 to a_C ($a_C = a_4$ for machinery A, $a_C = a_6$ for machinery B, $a_C = a_5$ for machinery C, $a_C = a_6$ for machinery D).

The interval values in the ON and EP columns are the values of subjective estimates of the operation noise and environmental pollution projected to a scale ⟨0, 100⟩ by a machinery expert, where 0 is the best desirable result. These data perform the first stage of the three-stage fuzzy system of multicriteria evaluation of alternatives (see Fig. 1).

5.2 Results

At the first-stage of the model in Fig. 1 it is not obvious from the data of Table 2 what machinery is optimal from the economic, social and ecological point view. Thus, the data are used for the calculation of the interval ⟨ae_{min}, ae_{max}⟩ of the annual expenditure equivalent (AE) for each machinery A, B, C, D. The *ae* value calculations require to determine the value of cost of capital of the project; the authors with the cooperation of experts estimate it at the 10 % p.a. The upper limit of *ae* acceptability is given by 50 thousand EUR. For the annual expenditure equivalent (AE) it applies (Han et al., 2014):

$$\mathrm{AE} = \mathrm{NPV}/\mathrm{A}_{t*,i}, \text{where } \mathrm{NPV} = \sum_{t=0}^{t*} \mathrm{CF}_t / (1+i)^t \text{ and } \mathrm{A}_{t*,i} = \sum_{t=1}^{t*} 1/(1+i)^t \tag{2}$$

where NPV is the net present value of annual expenditures, CF_t is the annual expenditure at the time t = 1…t* and i is a discount rate at the level of the cost of capital of the project.
From (2) we calculate the AE interval limits for each machinery.
For the machinery A it applies:

- $ae_{min} = (31 + 7 / 1.1 + 11 / 1.1^2 + 16 / 1.1^3 + 24 / 1.1^4) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4) = 23.61$,
- $ae_{max} = (31 + 9 / 1.1 + 14 / 1.1^2 + 20 / 1.1^3 + 30 / 1.1^4) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4) = 27.14$.

For the machinery B it applies:

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- ae_{min} = (42.5 + 5 / 1.1 + 8 / 1.1^2 + 12 / 1.1^3 + 16 / 1.1^4 + 20 / 1.1^5 + 24 / 1.1^6) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5 + 1 / 1.1^6) = 22.86,
- ae_{max} = (42.5 + 6 / 1.1 + 10 / 1.1^2 + 15 / 1.1^3 + 20 / 1.1^4 + 25 / 1.1^5 + 30 / 1.1^6) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5 + 1 / 1.1^6) = 26.09.

For the machinery C it applies:

- ae_{min} = (57.6 + 4 / 1.1 + 7 / 1.1^2 + 10 / 1.1^3 + 14 / 1.1^4 + 19 / 1.1^5 / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5) = 25.3,
- ae_{max} = (57.6 + 5 / 1.1 + 9 / 1.1^2 + 12 / 1.1^3 + 18 / 1.1^4 + 22 / 1.1^5) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5) = 27.58.

And for the machinery D it applies:

- ae_{min} = (62.3 + 3 / 1.1 + 6 / 1.1^2 + 9 / 1.1^3 + 12 / 1.1^4 + 16 / 1.1^5 + 23 / 1.1^6) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5 + 1 / 1.1^6) = 24.63,
- ae_{max} = (62.3 + 4 / 1.1 + 8 / 1.1^2 + 11 / 1.1^3 + 14 / 1.1^4 + 20 / 1.1^5 + 31 / 1.1^6) / (1 / 1.1 + 1 / 1.1^2 + 1 / 1.1^3 + 1 / 1.1^4 + 1 / 1.1^5 + 1 / 1.1^6) = 27.62.

The resulting limits of AE, ON and EP are summarized in Table 3. They perform the data at the second- stage of the model in Fig. 1. As we see, machinery B shows the best outcomes in terms of the economic criterion AE. However, the criteria ON and EP rank it as the worst variant.

Table 3. Intervals of possible values of annual expenditure equivalent (AE) in thousands of EUR, machine operation noise (ON) and environmental pollution (EP) of machinery A, B, C, D

Variant / Criterion	**AE**	**ON**	**EP**
Machinery A	⟨23.61, 27.14⟩	⟨20, 40⟩	⟨50, 70⟩
Machinery B	⟨22.86, 26.09⟩	⟨60, 70⟩	⟨60, 80⟩
Machinery C	⟨25.3, 27.58⟩	⟨30, 50⟩	⟨40, 60⟩
Machinery D	⟨24.63, 27.62⟩	⟨60, 70⟩	⟨50, 70⟩

Source: own processing

The problem is thus going to be resolved at the third stage of the model – in block *K* (see Fig. 1 on the right).

The block *K* is entered by the numerical vectors (*ae*, *on*, *ep*), $ae \in \langle 0, 50\rangle$, $on \in \langle 0, 100\rangle$, $ep \in \langle 0, 100\rangle$ of the triple of linguistic variables AE, ON and EP with terms $\underline{L}_i$, $\underline{M}_i$ and $\underline{H}_i$, i = ae, on, ep. Let us assume their fuzzification tables are symmetric and, except for the indices, they are identical (i.e., for all i = ae, on, ep it applies: $a_i = 20$, $b_i = 40$, $c_i = 60$, $d_i = 80$).

From the block *K* the number $v \in \langle 0, 100\rangle$ of the linguistic variable SUI is derived with the terms $\underline{L}$, $\underline{M}$ and $\underline{H}$ that has a character of sustainable economic, social and environmental costs. The set of inference rules of the type (($\underline{T}_{ae}$, $\underline{T}_{on}$, $\underline{T}_{ep}$), $\underline{T}$), $\underline{T}_i \in \{\underline{L}_i, \underline{M}_i, \underline{H}_i\}$, $\underline{T} \in \{\underline{L}, \underline{M}, \underline{H}\}$ has 3^3 elements formed by the strategy of the predominant element. This strategy assigns to the given left side of the rule the very term $\underline{T}$, which prevails on the left side. If there is no such a prevailing term $\underline{T}$, the term $\underline{M}$ is chosen.

By converting the coordinates of the numerical vector x_i into a scale in the range 0 to 100 we get the recalculated vectors (u_1, u_2, u_3), for which it applies: $u_1 = 100 \cdot ae / 50 = 2 \cdot ae$, $u_2 = on$ and $u_3 = ep$. In the general case, to obtain the outputs v_{min} and v_{max}, it would be necessary for all machineries to calculate the values of *v* to the eight recalculated vectors of all combinations of the interval limits of uncertain values. In this case the combination (u_{1min}, u_{2min}, u_{3min}) for v_{min} and (u_{1max}, u_{2max}, u_{3max}) for v_{max} is fully sufficient.

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The flowchart of the fuzzy algorithm (see Fig. 4) leads to a system of equations of the class $\mu = \mu_{VI}$ (stated above the Fig. 4) based on which the results are calculated. The results v_{min}, v_{max} of the fuzzy procedure in the bock K and arithmetic mean y as SUI index are summarized in Table 4.

Table 4. The results v_{min}, v_{max} of the fuzzy procedure in the block K and y of analysed machineries representing the SUI index

Machinery A: • v_{min}: $h(u_{1min}) = h(47.22) = \{\underline{M}_{ae}\}$, $h(u_{2min}) = h(20) = \{\underline{L}_{on}\}$, $h(u_{3min}) = h(50) = \{\underline{M}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{L}_{on}, \underline{M}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 1\}\} = 1$, $\mu = \mu_{VI}$, $\mathbf{v_{min} = 2000 / 40 = 50}$. • v_{max}: $h(u_{1max}) = h(54.28) = \{\underline{M}_{ae}\}$, $h(u_{2max}) = h(40) = \{\underline{M}_{on}\}$, $h(u_{3max}) = h(70) = \{\underline{M}_{ep}, \underline{H}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})$, $(\underline{M}_{ae}, \underline{M}_{on}, \underline{H}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 0.5\}, \{\min\{1, 1, 0{,}5\}\} = \max\{0.5, 0.5\} = 0.5$, $\mu = \mu_{VI}$, $\mathbf{v_{max} = 1250 / 25 = 50}$. • $\mathbf{y_A = (50 + 50) / 2 = 50}$.
Machinery B: • v_{min}: $h(u_{1min}) = h(45.72) = \{\underline{M}_{ae}\}$, $h(u_{2min}) = h(60) = \{\underline{M}_{on}\}$, $h(u_{3min}) = h(60) = \{\underline{M}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 1\}\} = 1$, $\mu = \mu_{VI}$, $\mathbf{v_{min} = 2000 / 40 = 50}$. • v_{max}: $h(u_{1max}) = h(52.18) = \{\underline{M}_{ae}\}$, $h(u_{2max}) = h(70) = \{\underline{M}_{on}, \underline{H}_{on}\}$, $h(u_{3max}) = h(80) = \{\underline{H}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{H}_{ep})$, $(\underline{M}_{ae}, \underline{H}_{on}, \underline{H}_{ep})\}$, $M_{\underline{L}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 0.5, 1\}\} = \max\{0.5\} = 0.5$, $M_{\underline{H}} = \max\{\min\{1, 0.5, 1\}\} = \max\{0.5\} = 0.5$, $\mu = \mu_{VI}$, $\mathbf{v_{max} = 2341.66 / 37.5 = 62.44}$. • $\mathbf{y_B = (50 + 62.44) / 2 = 56.22}$.
Machinery C: • v_{min}: $h(u_{1min}) = h(50.6) = \{\underline{M}_{ae}\}$, $h(u_{2min}) = h(30) = \{\underline{L}_{on}, \underline{M}_{on}\}$, $h(u_{3min}) = h(40) = \{\underline{M}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{L}_{on}, \underline{M}_{ep})$, $(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 1\}\} = 1$, $\mu = \mu_{VI}$, $\mathbf{v_{min} = 2000 / 40 = 50}$. • v_{max}: $h(u_{1max}) = h(55.16) = \{\underline{M}_{ae}\}$, $h(u_{2max}) = h(50) = \{\underline{M}_{on}\}$, $h(u_{3max}) = h(60) = \{\underline{M}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 1\}\} = 1$, $\mu = \mu_{VI}$, $\mathbf{v_{max} = 2000 / 40 = 50}$. • $\mathbf{y_C = (50 + 50) / 2 = 50}$.
Machinery D: • v_{min}: $h(u_{1min}) = h(49.26) = \{\underline{M}_{ae}\}$, $h(u_{2min}) = h(60) = \{\underline{M}_{on}\}$, $h(u_{3min}) = h(50) = \{\underline{M}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 1\}\} = 1$, $\mu = \mu_{VI}$, $\mathbf{v_{min} = 2000 / 40 = 50}$. • v_{max}: $h(u_{1max}) = h(55.24) = \{\underline{M}_{ae}\}$, $h(u_{2max}) = h(70) = \{\underline{M}_{on}, \underline{H}_{on}\}$, $h(u_{3max}) = h(70) = \{\underline{M}_{ep}, \underline{H}_{ep}\}$, $H = \{(\underline{M}_{ae}, \underline{M}_{on}, \underline{M}_{ep})$, $(\underline{M}_{ae}, \underline{H}_{on}, \underline{H}_{ep})\}$, $M_{\underline{L}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 0.5, 0.5\}\} = \max\{0.5\} = 0.5$, $M_{\underline{H}} = \max\{\min\{1, 0.5, 0.5\}\} = \max\{0.5\} = 0.5$, $\mu = \mu_{VI}$, $\mathbf{v_{max} = 2341.66 / 37.5 = 62.44}$. • $\mathbf{y_B = (50 + 62.44) / 2 = 56.22}$.

Source: own processing

Since $y_A = y_C < y_B = y_D$, machineries A and C should be preferred to B and D.

6. Discussion

We applied the three-stage fuzzy model that sophisticatedly projected the results of partial criteria AE, ON and EP into the value y representing the here introduced "Sustainability Utility Index" (SUI). The alternatives were given by four equally powerful building machineries differentiating in their lengths of economic life, expenditure and cost characteristics, and the vague characteristics of traffic noise and environmental pollution.

Regarding the RQ1 "Does the fuzzy model of multi-criteria evaluation have the property to state a rational decision from the point of view of stakeholders and principals of sustainability?" we state that the used version of the fuzzy model is optimally set to solve multicriteria problems under conditions of uncertainty and different characteristics of input variables. The result of the fuzzy process allows the stakeholders to take optimal decisions in view of financial and sustainability criteria. Specifically in this study, SUI measured by y points to the optimal choice from a set of considered alternatives, which is $y_A = y_C < y_B = y_D$. Machineries A and C should be investment-preferred as their complex contribution to the cost minimalization and sustainability pursue is better.

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The specifics of the fuzzy method allow the users to consider and process inputs of diverse nature and thus enable a wide application when deciding on the optimal choice and setting properties of machinery from the operational point of view and on the industrial investment management strategy. The first aspect was confirmed by Luo et al. (2020) when developing one-tail confidence-interval-based fuzzy testing method to evaluate quality characteristic of machinery while considering statistical parameters of process management. From the strategic point of view good results were achieved by Ghorabaee et al. (2018) who applied the hybrid approach based on the fuzzy extension of Stepwise Weight Assessment Ratio Analysis and Criteria Importance Through Inter Criteria Correlation methods for sustainability assessment of construction equipment. Ayağ & Özdemir (2006) understood the need to support the sustainable development of the manufacturing industry with regards to various uncertainties of waste components (Hašková et al. 2019). They proposed fuzzy comprehensive evaluation method based on the process of analytical hierarchy of fuzzy extension.

Generally, we can state that the fuzzy methods differ in the degree of complexity of the computational process and the volume of reflected data (Skare et al. 2023a,b). The first point is a consequence of different versions of fuzzy approaches used, the second point is given by the characteristics of the problem and the scope.

Regarding RQ2 our presented inference fuzzy system most resembles the Mamdani´s inference system. The difference between Mamdani's and our approach is that Mamdani´s rules are based on fuzzy implications (i.e., deductively invalid modus ponens judgments), while rules used by our approach are based on the "principle of extension" (an n-dimensional case of image induction). Therefore, our system avoids several complications associated with the semantics of fuzzy implications in various fuzzy logics, it is more "user friendly" from the point of view of a manager and a user and it is easily algorithmized. Unlike Mamdani´s and other fuzzy systems, whose behavior is derived from mathematical models of open-loop process control (Aksjonov et al., 2020; Camacho et al., 2021), this fuzzy system mimics the processes ongoing when solving problems intuitively and deductively in the human mind.

Conclusions

The sustainability measures (economic, environmental, and social) were treated within the multicriteria ex-ante evaluation of alternatives of production systems in terms of uncertainty of input data. Sustainability criteria were expressed by sustainable parameters of a vague nature.

The professional and scientific literature describes many methods that are used to solve multicriteria decision-making problems under vague inputs and uncertain conditions. The fuzzy approach suits this type of tasks very well. Compared to classical methods, its main advantage is that it approaches to human thinking and the way people express themselves. Thus, it is possible to model the meaning of words and expressions using the theory of fuzzy sets. As a result, fuzzy systems achieve more realistic outcomes than conventional systems.

The novelty of the paper is a construction of the three-stage model of the fuzzy assessment of a sustainable production in terms of its cost indicators and vague sustainable characteristics. The third step of the model is a computational algorithm the "fuzzy processor" the core of which is introduced in methodology. The fuzzy technique was applied to the optimal solution of a building machinery selection based on the cost and vague input parameters.

We see two-fold limits of the fuzzy system application: 1) on the general level the fuzzy systems lack the ability to learn and memorize 2) determining or tuning a suitable membership function and fuzzy rules is not always clear; an extensive testing often fails to say how many membership functions are needed.

The strengths of our system are its intelligible and computationally undemanding procedure, which helps the managers when solving problems with sharp and vague inputs considering the maximum uncertainty in the occurrence of linguistic variables considered. This fuzzy application provides managers with a faster, more accurate and more user-friendly method in comparison with methods based on analytical description. Thus, it aims to optimize the manager's decision, who minimizes the investment and operation costs of the building machine. The future challenge is to acquaint potential users with this computational tool. The aim of the paper both on the theoretical and application level specified by two RQ was fulfilled.

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